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Crossover exponent for piecewise directed walk adsorption on Sierpinski fractals

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Abstract. We study the problem of critical adsorption of piecewise directed random walks on a boundary of fractal lattices that belong to the Sierpinski gasket family. By applying the exact real space renormalization group method, we calculate the crossover exponent ϕ, associated with the number of adsorbed steps, for the complete fractal family. We demonstrate that our results are very close to the results obtained for ordinary self-avoiding walk, and discuss the asymptotic behaviour of ϕ at the fractal to Euclidean lattice crossover.

1. Introduction

It is well known that a long flexible linear polymer in a good solvent with an attractive short-range force between the polymer and the container wall undergoes an adsorption transition [1–3]. A good model for this phenomenon is a self-avoiding random walk (SAW) on some lattice with an adsorbing surface (boundary). In this model each monomer (step) in the bulk has a Boltzmann weight x, whereas the interaction with the adsorbing wall is taken into account by assigning an energy $\varepsilon_w < 0$ to each monomer that is found at the surface. For temperatures T higher than the critical temperature T_a of the adsorption, the polymer is in a desorbed phase, and for $T < T_a$ it is in an adsorbed phase. At the adsorption transition the number M of adsorbed monomers scales with the total number N of monomers as $M \sim N^{\phi}$, where ϕ is the crossover exponent.

The above model has been widely studied on various lattices and via a number of techniques ([3] and references therein). For two-dimensional Euclidean lattices exact results were found through conformal field theory [4–6] and much numerical work was completed using Monte Carlo [7, 8], exact enumeration [9–13], transfer matrix [14, 15], series expansion [16] and renormalization group [17, 18] techniques. Bouchaud and Vannimenus [19] developed a real space renormalization group (RSRG) approach to study the critical adsorption of SAW on finitely ramified fractal lattices (which may serve as crude models for real amorphous materials). In addition to some other results, these authors found the exact values of the crossover exponent ϕ for the case of two- and three-dimensional Sierpinski gaskets (SG).

The RSRG method [19] was applied on other fractals, in particular on other members of the two-dimensional SG fractal family [20], in order to find out how the crossover exponent changes when properties of the fractal are systematically changed. Members of this family can be

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